The University of Alabama at Birmingham (UAB)
Department of Physics

PH 461/561 – Classical Mechanics I – Fall 2005

<u>Assignment # 5</u> Due: **Thursday, September 15**
(Turn in for credit!)

<u>Activities based on previous lectures:</u>

1. A particle of mass *m* moves in one dimension under the force:

$$F(x) = 2aA\left(e^{-2ax} - e^{-ax}\right) \quad \text{where } a > 0,\ A > 0$$

a) Discuss the effect of this force on the total mechanical energy of the particle.
b) Find an expression for the potential energy *V(x)* of the particle.
c) Graph this potential energy by hand (Computer use for checking ok)
d) Provide a qualitative discussion of the possible types of motion depending on the total energy *E*. (bound, unbound motion, turning points, etc.)
e) Find the position of the point of equilibrium.
f) For total energy slightly above $-A$, the motion of the particle is periodic and the potential energy may be approximated by parabolic well. In order to prove this perform the following steps

f-1) Expand $V(x)$ in a power series around $x = 0$ keeping only terms up to second order.

f-2) Calculate the period of small oscillations around the point of equilibrium (i.e., the minimum of the potential energy).

Hint: $e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{2\cdot 3} + \frac{x^4}{2\cdot 3\cdot 4} + \cdots$

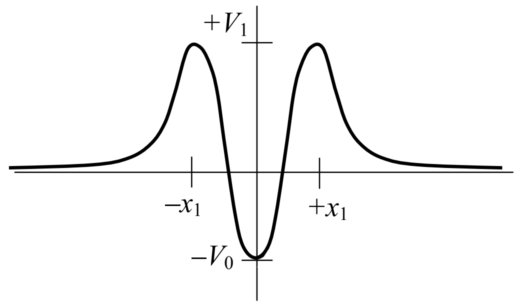
2. A particle of mass *m* is acted on by a force whose potential energy is

$$V(x) = ax^2 - bx^3\ ;\ a, b \text{ positive constants}$$

a) Graph this potential energy by hand (Computer use for checking ok)
b) Find the force
c) Discuss the effect of this force on the total mechanical energy of the particle.
d) Provide a qualitative discussion of the possible types of motion depending on the total energy *E*. (bound, unbound motion, turning points, etc.)
e) Assume that the particle starts at the origin $x = 0$ with velocity v_0. Show that if $|v_0| < v_c$, where v_c is a certain critical velocity, the particle will remain confined to a region near the origin. Find v_c.

3. An alpha particle in a nucleus is held by a potential having the shape shown in Fig. 1 below.

   a) Describe the kinds of motion that are possible.
   b) Devise a function $V(x)$ having this general form and having the values $-V_0$ and V_1 at $x=0$ and $x=\pm x_1$
   c) Find the corresponding force.



4. A particle of mass m is subject to a force

$$F(x) = -kx + \frac{kx^3}{a^2}; \; k, a \text{ positive constants}$$

   a) Find $V(x)$ and discuss the kinds of motion which can occur.
   b) Show that if $E = \frac{1}{4}ka^2$ the "energy integral" discussed in class can be evaluated by elementary methods. Find $x(t)$ for this case, choosing x_0, t_0 in any convenient way. Show that your result agrees with the qualitative discussion in part (a) for this particular energy.

5. A particle of mass m is repelled from the origin by a force inversely proportional to the cube of its distance from the origin.
   a) Discuss the effect of this force on the total mechanical energy of the particle.
   b) Find an expression for the potential energy $V(x)$ of the particle.
   c) Graph this potential energy by hand (Computer use for checking ok)
   d) Set up and solve the equation of motion if the particle is initially at rest at a distance x_0 from the origin.

6. A particle of mass m is subject to a force

$$F = -kx + \frac{a}{x^3}; \qquad k, a \text{ positive constants}$$

   a) Find the potential energy $V(x)$ of the particle.
   b) Describe the nature of the solutions, and find the solution $x(t)$.
   c) Provide an interpretation of the motion when $E^2 >> ka$

7. A particle of mass *m* is subject to a force given by

$$F = B\left(\frac{a^2}{x^2} - \frac{28a^5}{x^5} + \frac{27a^8}{x^8}\right); \qquad B, a \text{ positive constants}$$

The particle moves only along the positive *x*-axis

a) Find and sketch the potential energy.
b) Describe the types of motion which may occur. Locate all equilibrium points and determine the frequency of small oscillations about any which are stable.
c) A particle starts at $x = \frac{3a}{2}$ with a velocity $v = -v_0$, where v_0 is positive. What is the smallest value of v_0 for which the particle may eventually escape to a very large distance? Describe the motion in that case. What is the maximum velocity the particle will have? What velocity will it have when it is very far from its starting point?

8. The potential energy for the force between two atoms in a diatomic molecule has the approximate form:

$$V(x) = -\frac{a}{x^6} + \frac{b}{x^{12}}; \qquad a, b \text{ positive constants}$$

where *x* is the distance between the atoms.

a) Find the force.
b) Assuming one of the atoms is very heavy and remains at rest while the other moves along a straight line, describe the possible motions.
c) Find the equilibrium distance and the period of small oscillations about the equilibrium position if the mass of the lighter atom is *m*.

9. Find the solution for the motion of a body subject to t linear repelling force $F = kx$. Show that this is the type of motion to be expected in the neighborhood of a point of unstable equilibrium.

10. Fowles & Cassiday, 7th Edition, **Problem 2.14.**